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THE MSSM ON THE INTERVAL

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We review electroweak symmetry breaking in supersymmetric models with a compact fifth dimension, the interval. We show how boundary conditions for hypermultiplets can be obtained dynamically by brane mass terms, and present formulae for the spectrum in the presence of general bulk mass matrices. After giving a brief overview on the literature of models, we describe in detail a recently proposed model that at energies below the compactification scale reduces to the MSSM with a very peculiar superpartner spectrum.

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1. Introduction

Supersymmetric models in five dimensions with a low compactification scale (i.e. TeV or multi TeV) have received considerable interest in recent years.^{1–18} The four-dimensional supersymmetric Standard Model (MSSM) naturally incorporates electroweak symmetry breaking (EWSB), as loops of the top/stop sector drive the Higgs mass squared to negative values, thus enforcing the Higgs to acquire a nontrivial vacuum expectation value (VEV). The size of the electroweak scale is controlled by the scale of supersymmetry (SUSY) breaking, in particular the mass of the stop itself. The quadratic sensitivity of the electroweak scale to the ultraviolet physics is cut off above the stop mass, and as long as the latter does not exceed several TeV, EWSB can occur in a fairly natural manner. In fact, understanding the mechanism that triggers SUSY breaking is one of the main issues in supersymmetric theories, and it should determine the phenomenology of supersymmetric particles at future high-energy colliders as the LHC.

The standard way of breaking SUSY in models with Extra Dimensions is the Scherk-Schwarz (SS) mechanism.^{19,20} This breaking is unique to Extra Dimensions, as it preserves different higher dimensional supersymmetries at different points in the Extra Dimension(s). For instance, in five dimensional (5d) models compactified on an interval of length πR , $N = 2$ SUSY is preserved in the bulk, while at the two

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boundaries two different $N = 1$ supercharges survive, resulting in fully broken SUSY in the 4d effective theory. SS-breaking exhibits very little UV sensitivity, due to the fact that it is a nonlocal mechanism, and hence soft mass terms corresponding to local counterterms in the 5d bulk or 4d brane actions are not allowed. As a result, all radiative effects related to SUSY breaking are cut off at the compactification scale. The only UV sensitivity thus results from the supersymmetric renormalization of the bulk and brane operators, i.e. a supersymmetric running of the gauge and Yukawa couplings. While the gauge coupling is always linearly divergent, the UV-sensitivity of the Yukawa terms depend on the nature of the chiral superfields involved. SUSY forces the Yukawa interactions to be localized on the boundary, nevertheless they can involve bulk fields. Simple dimensional analysis shows that the more bulk fields enter a Yukawa interaction the more sensitive the theory is to unknown UV physics, as wave function renormalization of bulk fields are linearly divergent. Yukawa interactions with only bulk fields also increase the number of free parameters of that model, as one can write different couplings at the two boundaries. While from these considerations it seems preferable to localize all matter and Higgs multiplets at the boundaries, it turns out that EWSB does not take place in such a setup.¹³ One is lead to consider some matter or Higgs fields to live in the bulk. In this case, particular care is needed in choosing the boundary conditions for these fields in order to avoid generation of quadratically divergent Fayet Iliopoulos (FI) terms at the branes.²¹

In this paper we will review several supersymmetric models of EWSB with a fifth dimension compactified on a (flat) interval of length πR . Such models can be classified according to where Higgs and matter sectors live. Fields that live in the bulk must transform as hypermultiplets of the extended bulk supersymmetry. In Sec. 2 we review the description in terms of $N = 1$ multiplets, relate consistent boundary conditions dynamically to appropriate brane actions, and compute mass spectra in the presence of general bulk mass matrices. Successful EWSB forbids both matter and Higgs sectors to be completely localized. In Sec. 3 we review some of the models that achieve EWSB by delocalizing either the Higgs fields or some of the matter fields. In Sec. 4 we then focus on a recently proposed model of a quasilocalized Higgs sector, and describe in some detail its experimental signatures, in particular its distinctive pattern of squark and slepton masses.

2. Supersymmetry on the Interval

Let us write the general Lagrangian of a bulk hypermultiplet in terms of $N = 1$ chiral superfields. The 5d vector multiplet splits into a 4d one V and a chiral adjoint multiplet Σ , while the hypermultiplet is made up out of two chiral multiplets \mathcal{H} and

\mathcal{H}^c . The Lagrangian can then be expressed as ^{22,23,24}

$$\mathcal{L}^{\text{hyper}} = \int d^4\theta \frac{\mathcal{T} + \bar{\mathcal{T}}}{2} \left\{ \bar{\mathcal{H}} \exp(T_a V^a) \mathcal{H} + \mathcal{H}^c \exp(-T_a V^a) \bar{\mathcal{H}}^c \right\} \\ - \int d^2\theta \left\{ \mathcal{H}^c (\overleftrightarrow{\partial}_y - \mathcal{M}\mathcal{T} + T_a \Sigma^a) \mathcal{H} + h.c. \right\}, \quad (1)$$

where the $\overleftrightarrow{\partial} = \frac{1}{2}(\overleftarrow{\partial} - \overrightarrow{\partial})$. The mass matrix \mathcal{M} is hermitian and acts on the internal flavor indices of the hypermultiplet. The radion field \mathcal{T} will be taken nondynamical,

$$\mathcal{T} = R + 2\omega\theta^2. \quad (2)$$

Its scalar component parametrizes the size of the extra dimension and a non-zero ω implements the SS breaking.^{22,25,26}

Let us start with a single hypermultiplet, in which case the hermitian matrix \mathcal{M} is a real number that we call M' . The boundary conditions can be obtained dynamically from the action principle by adding a suitable supersymmetric brane mass term at $y_f = 0, \pi$

$$\mathcal{L}_f^{\text{hyper}} = -\frac{1}{2} \int d^2\theta r_f \mathcal{H}^c \mathcal{H} + h.c., \quad (3)$$

with $r_f^2 = 1$.¹⁸ The boundary conditions following from varying the action can be given in superfield form:

$$(1 - r_f)\mathcal{H} = 0, \quad (1 + r_f)\mathcal{H}^c = 0. \quad (4)$$

As $r_f = \pm 1$, only one of the two equations in (4) is non-trivial, giving Dirichlet boundary conditions to one chiral multiplet, while the other superfield remains unconstrained.^a In the orbifold picture the quantities r_f are known as the parities of the field \mathcal{H} (w.r.t. the fixed point at $y = y_f$). The scalar boundary conditions follow once the auxiliary fields are integrated out:

$$(1 - r_f)H = 0, \quad (1 + r_f)(\partial_y - M')H = 0, \quad (5)$$

$$(1 + r_f)H^c = 0, \quad (1 - r_f)(\partial_y + M')H^c = 0. \quad (6)$$

For $r_0 = -r_\pi = \pm 1$ one finds for the scalar mass eigenvalues

$$\left(1 + \frac{M'^2}{\Omega^2}\right) \sin^2(\Omega\pi R) = \sin^2(\pi\omega), \quad (7)$$

where we have defined $\Omega^2 = m^2 - M'^2$. In the case $r_0 = r_\pi = \pm 1$ one finds

$$\left(\cos(\pi\Omega R) \pm \frac{M'}{\Omega} \sin(\pi\Omega R)\right)^2 = \sin^2(\pi\omega). \quad (8)$$

The fermionic spectrum is obtained by setting the SS-parameter $\omega = 0$.

^aThe case of gauge multiplets can be treated analogously, see Refs. 27, 28.

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A single hypermultiplet with these boundary conditions is known to produce quadratically divergent FI terms for the hypercharge, localized at the boundaries.²¹ Although they can be absorbed by a shift in the field Σ_Y , they reappear as mass terms for charged fields due to the Yukawa coupling in Eq. (1). One concludes that the mass term M' is effectively renormalized and in particular quadratically divergent. This can be avoided if two hypermultiplets of the same kind but with “orthogonal” boundary conditions are considered. To this end, one considers the two hypermultiplets to form a doublet under a formal $SU(2)_H$ global symmetry.^b The brane Lagrangian

$$\mathcal{L}_f^{\text{hyper}} = \frac{1}{2} \int d^2\theta \mathcal{H}^c \vec{r}_f \cdot \vec{\sigma} H + h.c., \quad (9)$$

produces the superfield boundary conditions

$$(1 + \vec{r}_f \cdot \vec{\sigma})\mathcal{H} = 0, \quad \mathcal{H}^c(1 - \vec{r}_f \cdot \vec{\sigma}) = 0, \quad (10)$$

and quadratically divergent FI terms are shown to be absent. The two fields in Eq. (10) that do not vanish at a given boundary carry opposite hypercharge and can have the MSSM superpotential couplings of the Higgs fields to boundary matter. The misalignment of the two vectors \vec{r}_0 and \vec{r}_π can be translated into a SS-parameter for the $SU(2)_H$ symmetry, given by $\cos(2\pi\tilde{\omega}) = \vec{r}_0 \cdot \vec{r}_\pi$. The mass matrix is conveniently parametrized as

$$\mathcal{M} = M' + M \vec{p} \cdot \vec{\sigma}, \quad (11)$$

where \vec{p} is a unit vector and $M' \in \mathbb{R}$, $M \in \mathbb{R}_+$. Again by integrating out the auxiliary fields, one finds the bosonic boundary conditions

$$(1 + \vec{r}_f \cdot \vec{\sigma})H = 0, \quad (1 - \vec{r}_f \cdot \vec{\sigma})(\partial_y - M' + c_f M)H = 0, \quad (12)$$

$$H^c(1 - \vec{r}_f \cdot \vec{\sigma}) = 0, \quad H_c(\overleftarrow{\partial}_y + M' + c_f M)(1 + \vec{r}_f \cdot \vec{\sigma}) = 0, \quad (13)$$

with $c_f = \vec{r}_f \cdot \vec{p}$. As mentioned before, the mass term M' is equivalent to a boundary D -term. Although no quadratic renormalization of this operator occurs, there is a linear divergent contribution $\sim M'\Lambda$. We will thus mostly be interested in the case of $M' = 0$, in which case the mass eigenvalues are given by the zeroes of the two equations¹⁷

$$\left(\cos(\Omega\pi R) - \frac{c_0 M}{\Omega} \sin(\Omega\pi R) \right) \left(\cos(\Omega\pi R) + \frac{c_\pi M}{\Omega} \sin(\Omega\pi R) \right) = \cos^2(\omega \pm \tilde{\omega})\pi, \quad (14)$$

where now $\Omega^2 = m^2 - M^2$.

The generic Neumann boundary condition

$$(\partial_y + \Omega_f)\Phi|_{y=y_f} = 0 \quad (15)$$

^bWe call it a “formal” symmetry as it might be explicitly broken by bulk and/or brane mass terms as well as Yukawa interactions.

leads to wave functions exponentially decaying in the bulk whenever

$$\Omega_0 R\pi \gg 1, \quad \Omega_\pi R\pi \ll -1. \quad (16)$$

These states are known as quasilocalized states. Their mass eigenvalues follow from the spectra Eq. (7), (8) and (14) with $\Omega = i\Omega_f$, i.e.

$$m_f^2 = -\Omega_f^2 + M^2 + \mathcal{O}(\epsilon), \quad \epsilon = e^{-\Omega_f \pi R}. \quad (17)$$

The parameter ϵ serves as an order parameter describing the degree of quasi-localization of the corresponding state. The limit $\epsilon \rightarrow 0$ describes a fully localized chiral multiplet while at the same time the heavy KK modes (the ones with real Ω) decouple. Even for moderately large values of the bulk mass, e.g. $M \sim R^{-1}$, the localization is quite efficient and corrections to the masses Eq. (17) are strongly suppressed. This suggests some kind of systematic expansion in the parameter ϵ .

3. Previous Models

Models of EWSB in supersymmetric theories with TeV-size Extra Dimensions and SS-breaking can be classified according to where the matter and Higgs sectors live. Any bulk multiplet will feel SUSY breaking at the tree level, while brane fields can obtain soft masses only through their coupling to bulk fields (i.e. the gauge sector and possibly other matter fields) via loop effects. A somehow hybrid status is assumed by the quasi-localized states discussed in Sec. 2. As can be seen from Eq. (17), the leading contribution (which becomes exact in the strictly localized limit) is independent of the SS parameter ω , and all SUSY breaking effects are controlled by the small parameter ϵ .

In the limit of unbroken SUSY, EWSB cannot take place. With a fully localized Higgs, the leading contribution to the soft squared masses is provided by loops of the electroweak gauge sector, and hence it is of order α_W . This contribution is positive and cannot trigger EWSB. The usual negative stop/top contributions are however weakened by the fact that the stop masses are itself a one loop effect (predominantly generated by gluon/gluino loops) and, hence, are effectively two loop contributions $\sim \alpha_t \alpha_s$. As has been shown in Ref. 13, this contribution is too weak for EWSB to occur. There are essentially two ways out of this dilemma: either the top/stop sector or the Higgs sector is taken to propagate in the bulk. In the former case the stops feel SUSY breaking at tree level and the negative contribution to the Higgs squared mass is enhanced, in the latter case the Higgs soft scalar mass matrix can possess vanishing or even tachyonic eigenvalues. We should mention that most models do not yield the MSSM at low energies. This is due to the fact that the SUSY-breaking and compactification scales are the same, and unless the SS-parameter is taken to very small values, there is no regime where the theory can be formulated as a 4d supersymmetric Standard Model with soft breaking terms. In other words, there is no mass gap between the superpartners and the KK-partners.

The scenario with a delocalized Higgs sector is realized, for instance, in the model of Refs. 2–4. Evaluating Eq. (14) at $\omega = 0$, we see that the supersymmetric Higgs

sector consists of a tower of chiral superfields H_u^m and H_d^m with masses $m = |n \pm \tilde{\omega}|$ with $n \geq 0$ integer.^c SS-SUSY breaking splits the scalar masses of each level as $|n \pm \tilde{\omega} \pm \omega|$, with the corresponding eigenstates $h_u^m \pm h_d^{m\dagger}$. A massless mode can be achieved at tree level by choosing $\omega = \tilde{\omega}$, which actually constitutes a flat direction of the tree level potential. As has been shown in Ref. 4, radiative corrections can then successfully trigger EWSB. Had the Higgs been localized, the tree level masses would be controlled by the μ -parameter and EWSB would be impossible to achieve. Due to the fact that the massless mode is a flat direction at tree level, the Higgs mass turns out to be rather light ($\lesssim 110$ GeV for the most favourable value of ω). The most efficient way to raise the Higgs mass is thus to delocalize the matter sector, or parts of it.

A model of this kind has been proposed in Ref. 6, albeit with a quite different Higgs sector that contains only one Higgs doublet. The reason why this works is that one can chose the boundary conditions such that an up-type Higgs survives at $y = 0$ and a down-type one at $y = \pi R$: Choosing $r_0 = 1$, $r_\pi = -1$ in Eq. (4) leaves \mathcal{H} and \mathcal{H}_c at $y = 0$ and $y = \pi R$ respectively. Assigning $Y = \frac{1}{2}$ to the whole hypermultiplet, one can write the up type Yukawa couplings at $y = 0$ and the down type ones at $y = \pi R$. In the matter sector, this requires at least the electroweak doublets to propagate in the bulk. The spectrum for the Higgs sector is given by Eq. (8) with $M' = 0$. The supersymmetric spectrum thus consists of chiral superfields H_u^m and H_d^m with m taking half-integer values. In Ref. 6 the value $\omega = \frac{1}{2}$ was chosen, which again leaves a massless Higgs field at tree level. As has been pointed out,²¹ although there is no zero mode anomaly in this model, there do appear localized anomalies at the two boundaries. This anomalies can be canceled by the introduction of a Chern-Simons term in the bulk. However, along with these anomalies, quadratically divergent FI terms are generated at the branes. As mentioned in Sec. 2, they can be removed by a redefinition of the field Σ^Y , which however leads to UV-sensitive bulk mass-terms and spontaneous quasi-localization.²¹ A careful analysis of this model, including the effects of arbitrary mass terms for the matter multiplets in the bulk was performed in Ref. 13, 14. Also, the effect of a second Higgs hypermultiplet, canceling the quadratic divergence, was incorporated. In the absence of any bulk masses, this Higgs sector is then equivalent to the model of Ref. 4 described in the previous paragraph with the special value $\omega = \frac{1}{2}$. It was found that if the matter sector is completely delocalized, the tachyonic contributions of the top/stop sector are too strong, thus resulting in unstable D -flat directions. This problem had previously been realized in Ref. 7. The authors suggested a nonrenormalizable quartic term in the superpotential in order to stabilize the potential at large VEVs. Another possibility is to move away from the value $\omega = 1/2$, as realized, e.g., in Ref. 8.^d On the other hand, it was also shown

^cWe give the masses in units of $1/R$ and without loss of generality always assume the twist parameters ω and $\tilde{\omega}$ to lie in the interval $[0, 1/2]$.

^dSee discussion of that model below.

that if the matter sector is completely localized (for instance, by assigning large bulk masses), EWSB does not take place, as the top/stop contributions are now two-loop and cannot overcome the positive one-loop electroweak ones.^e However, slightly delocalizing the top sufficiently enhances its one loop contribution to the Higgs mass and EWSB is found to be possible, with the degree of localization typically in the range $\epsilon \sim 5\%$ and the Higgs mass around $110 - 125$ GeV for compactification scales $1/R \sim 6 - 12$ TeV. The strong UV sensitivity of the Yukawa couplings in this model requires a rather low cutoff, $\Lambda \sim (2 - 3)M_c$,¹⁴ above which the top Yukawa coupling quickly becomes nonperturbative.

The scenario with a delocalized top/stop sector and localized Higgs doublets is realized, for instance, in the model of Ref. 8. In this model, the electroweak singlets were assumed to be the only bulk matter fields. Notice that the trace over the hypercharge of the electroweak singlets vanishes separately, so no FI term and, hence, no hidden UV sensitivity is generated. A μ parameter has to be introduced in the brane superpotential. Radiative EWSB is then triggered by the right handed top/stop sector. Taking μ and ω as free parameters of the model, $\tan\beta$ and the compactification scale are fixed by the minimization procedure. One typically finds a large value for $\tan\beta \sim 40$. For such large values of $\tan\beta$ and at values of $\mu \lesssim 600$ GeV, the SM-like Higgs turns out to be heavier than the non SM-like one, which has a mass approximately equal to the pseudoscalar mass, m_A . LEP bounds on the latter thus translate into lower bounds on μ , and in turn on the SM-like Higgs mass m_h . This leads to a rather heavy Higgs, with an ω -dependent lower bound of approximately $m_h \gtrsim 145$ GeV.

4. The MSSM with a Quasi Localized Higgs

In Sec. 2 we have pointed out that the relative UV insensitivity of the SS-mechanism can be preserved, even with a bulk Higgs sector, if the latter consists of two hypermultiplet doublets with brane and bulk masses appropriately chosen such as to produce boundary conditions that do not generate quadratically or linearly divergent boundary FI terms. The natural size for the bulk masses lies somewhere between the fundamental and the compactification scale, which in turn are typically separated by a factor $10 - 100$. Once the bulk mass exceeds the compactification scale, quasi localization of the lightest modes sets in quickly, while the heavy KK-modes decouple from the light spectrum. The minimal setup of this kind would have the matter sector completely localized. As pointed out before, an exact localization of the Higgs fields would not result in successful EWSB. However, for finite values

^eIt should be mentioned that the authors of Ref. 4 still find EWSB to occur at $\omega = 1/2$, although it results in a very small (and phenomenologically unacceptable) Higgs mass. For such a marginal breaking, the uncertainties in their large logarithm approximation are probably too significant to definitely decide whether EWSB takes place, and a precision two-loop calculation as in Ref. 14 is needed. On the other hand, for $\omega < 1/2$, EWSB is not marginal and the simplified treatment of Ref. 4 is justified.

of the localization parameter ϵ , the lightest Higgs modes do feel SS-SUSY breaking, as their wave function leaks into the bulk and is sensitive to the SUSY breaking boundary conditions at the distant brane. Clearly, this sensitivity can only be of order ϵ , as it must vanish in the limit of exact localization (i.e. $\epsilon \rightarrow 0$). In this section we will outline the model proposed in Ref. 18, which accomplishes EWSB in a very natural manner (i.e. without large fine-tuning), and leads to a quite unique superpartner spectrum. Due to the suppression of the SUSY breaking in the Higgs sector and the non-interference with the higher KK modes, a description in terms of the standard MSSM Higgs sector is possible at low energies, although it has to be borne in mind that above the compactification scale the theory is modified in an essential way and very different from the MSSM.

The reason why EWSB can work is that it is possible to generate tachyonic soft mass terms at tree level that are comparable in size with both the electroweak one-loop as well as the Yukawa controlled two-loop contributions. All of them are suppressed w.r.t. the compactification scale by loop factors or factors of ϵ . The only unsuppressed term is the supersymmetric mass term, i.e. the μ parameter. We choose boundary conditions of the form Eq. (10), as well as $M' = 0$. Note that this leaves one up and one down-type chiral Higgs superfield at each brane. In the limit of unbroken SUSY, there are two quasilocalized chiral multiplets with mass given by Eq. (17)

$$\mu^2 = (1 - c_0^2)M^2 + \mathcal{O}(\epsilon^2), \quad (18)$$

where $c_0 = \cos(2\pi\alpha_0)$ parametrizes the “angle” between the bulk and brane mass matrices, $c_0 = \vec{r}_0 \cdot \vec{p}$. The bulk mass scale M is of the order of the compactification scale or higher, i.e. in the multi TeV region. In order to get a μ -term of roughly the electroweak size and to avoid large cancellations, we require the angle α_0 or equivalently $s_0 = \sin(2\pi\alpha_0)$ to be small.

The fact that the μ -term and the soft terms arise at different orders in the ϵ expansion can be traced back to the following fact. Notice that both boundary and bulk mass matrices preserve $U(1)_H$ subgroups of the global $SU(2)_H$, generated by $\vec{r}_f \cdot \vec{\sigma}$ and $\vec{p} \cdot \vec{\sigma}$ respectively. For $\vec{r}_0 = \pm \vec{p}$ (corresponding to $s_0 = 0$) the surviving $U(1)$ at $y = 0$ and the $U(1)$ in the bulk coincide, this symmetry being broken only by the mismatched $U(1)$ at $y = \pi$. The zero modes feel this breaking through their wavefunctions, which are, however, suppressed at $y = \pi$ as $\sim \epsilon$. Hence we expect $\mu \sim \epsilon^2$ when $s_0 = 0$ as it can be checked from the ϵ expansion of fermionic mass eigenvalues. When $s_0 \neq 0$, the breaking of the $U(1)$ at $y = 0$ is really felt to $\mathcal{O}(1)$ as it occurs even for infinitesimally small $y > 0$ and hence the μ -term is unsuppressed. On the other hand, SUSY is broken à la Scherk-Schwarz, which can be interpreted as a mismatch of the surviving boundary $U(1)_R$ subgroups of the $N = 2$ $SU(2)_R$ automorphism group in the bulk. Again, the zero-mode wave-functions feel this only to $\mathcal{O}(\epsilon)$, and the corresponding soft terms are suppressed as $m^2 \sim \epsilon^2$.

The mass eigenvalues for broken SUSY can be inferred from Eq. (14). Since, as just explained, the soft terms are expected to be ϵ -suppressed with respect to M_c

and the KK-masses, there is a regime where a description in terms of the 4d MSSM is valid. Therefore for small ϵ the MSSM mass Lagrangian

$$\mathcal{L}_{\text{mass}} = -(\mu^2 + m_{H_u}^2) |H_u|^2 - (\mu^2 + m_{H_d}^2) |H_d|^2 + m_3^2 (H_u \cdot H_d + h.c.) \quad (19)$$

can be used. It is thus convenient to translate back the mass eigenvalues into the soft mass terms of Eq. (19). With the additional requirement $s_0 \sim \mathcal{O}(\epsilon)$ one finds¹⁸

$$m_{H_u}^2 = m_{H_d}^2 = 4M^2 \sin^2(\pi\omega)(1 - \tan^2(\pi\tilde{\omega})) \epsilon^2 + \dots, \quad (20)$$

$$m_3^2 = 4M^2 \sin(2\pi\omega) \tan(\pi\tilde{\omega}) \epsilon^2 + \dots, \quad (21)$$

while the μ -term is given by

$$\mu^2 = s_0^2 M^2 + \dots \quad (22)$$

Here, the ellipsis stands for terms suppressed by higher powers of ϵ and/or s_0 . To these tree level soft masses one has to add the radiative corrections. The squark masses will be dominated by the contribution from the gluinos, which is given by³

$$\Delta m_{\tilde{t}, \tilde{b}}^2 = \frac{2g_3^2}{3\pi^4} M_c^2 f(\omega), \quad (23)$$

where the function $f(\omega)$ is defined by

$$f(\omega) \equiv \sum_{k=1}^{\infty} \frac{\sin(\pi k \omega)^2}{k^3}. \quad (24)$$

Electroweak gauginos provide a radiative correction to the slepton and Higgs masses as

$$\Delta^{(1)} m_{H_u}^2 = \Delta^{(1)} m_{H_d}^2 = \frac{3g^2 + g'^2}{8\pi^4} M_c^2 f(\omega). \quad (25)$$

Furthermore there is a sizable two-loop contribution to the soft mass-terms of the Higgs, as well as to the quartic coupling, coming from top/stop loops with the one-loop generated squark masses given by Eq. (23). This contribution can be estimated in the large logarithm approximation by just plugging the one-loop squark masses in the one-loop effective potential generated by the top/stop sector.⁴ For the sake of this paper, where EWSB will not be marginal (as we will see later) it is enough to consider the effective potential in the large logarithm approximation, which yields the two-loop corrections to the Higgs masses

$$\Delta^{(2)} m_{H_u}^2 = \frac{3y_t^2}{8\pi^2} \Delta m_{\tilde{t}}^2 \log \frac{\Delta m_{\tilde{t}}^2}{Q^2}, \quad (26)$$

$$\Delta^{(2)} m_{H_d}^2 = \frac{3y_b^2}{8\pi^2} \Delta m_{\tilde{b}}^2 \log \frac{\Delta m_{\tilde{b}}^2}{Q^2}, \quad (27)$$

where the renormalization scale should be fixed to the scale of SUSY breaking, i.e. the gaugino mass ωM_c .⁴ Notice that the corrections from the bottom sector are also considered, which would only be relevant for large values of $\tan \beta$.

Finally, the leading two-loop corrections to the quartic self coupling of H_u and H_d in the potential

$$\Delta V_{\text{quartic}} = \Delta\gamma_u |H_u|^4 + \Delta\gamma_d |H_d|^4 \quad (28)$$

are given by

$$\Delta\gamma_u = \frac{3y_t^4}{16\pi^2} \log \frac{\Delta m_t^2 + m_t^2}{m_t^2}, \quad (29)$$

$$\Delta\gamma_d = \frac{3y_b^4}{16\pi^2} \log \frac{\Delta m_b^2 + m_b^2}{m_b^2}, \quad (30)$$

where m_t and m_b are the top and bottom quark masses respectively.

Electroweak symmetry breaking can now occur in our model in a very peculiar and interesting way. The tree-level squared soft masses m_{H_u, H_d}^2 given in Eq. (20) are suppressed by the factor ϵ^2 and therefore, for values of $M \sim M_c$ they can be comparable in size to the one-loop gauge corrections $\Delta^{(1)} m_{H_u, H_d}^2$ given by Eq. (25). Furthermore, the tree-level masses m_{H_u, H_d}^2 are negative for values of $\tilde{\omega} > 1/4$ and then there can be a (total or partial) cancellation between the tree-level and one-loop contributions to the Higgs masses. Under extreme conditions they can even cancel, $m_{H_u, H_d}^2 + \Delta^{(1)} m_{H_u, H_d}^2 \simeq 0$, in which case the negative two-loop corrections $\Delta^{(2)} m_{H_u}^2$ will easily trigger EWSB. On the other hand, in the limit of exact localization of the Higgs fields $\epsilon \rightarrow 0$ the tree-level masses will vanish and the one-loop gauge and two-loop top/stop corrections have to compete, which will make the EWSB marginal, as pointed out in Ref. 13, 14. These simple arguments prove that there is a wide region in the space of parameters $(\omega, \tilde{\omega}, \epsilon)$ where EWSB easily happens without much fine-tuning of these parameters.^f Of course EWSB also depends on the Higgsino mass μ and on the compactification scale M_c (or equivalently on the gluino mass as it happens in the MSSM) and we will be concerned about the possible fine-tuning in those mass parameters.

What other phenomenological requirements could possibly restrict the parameter space? One constraint arises if we require the existence of a stable Dark Matter (DM) particle. Note that gaugino masses are given by ωM_c , while Higgsinos are controlled by the much smaller μ -parameter. We thus expect the neutralino to be almost entirely Higgsino-like, with a mass basically given by μ . Clearly, the charged sleptons must be heavier than the Higgsinos. Their mass is controlled again by the gaugino mass ωM_c , although it is smaller by a loop factor. The μ -term also implicitly increases with M_c through the minimization conditions, but for smaller $\tilde{\omega}$ the absolute value of the soft mass terms in Eq. (20) decreases, which in turn allows for a smaller μ . The requirement that the neutralino be lighter than the charged sleptons thus favours the region $\omega > \tilde{\omega}$. As we will see below, a DM abundance consistent

^fA more thorough treatment of the fine tuning issue can be found in Ref. 18, where it was shown that the amount of fine tuning is 10% (4%) for $M_c = 6.6$ (10) TeV.

with recent WMAP results²⁹ points towards a rather large compactification scale ~ 50 TeV.

We can now solve the minimization conditions for a suitable values of $(M/M_c, \omega, \tilde{\omega})$, which will give us two predictions, $\tan\beta$ and μ as functions of the only left free parameter, M_c . We thus can compute the entire mass spectrum as a function of the compactification scale. The result is displayed in Fig. 1. In the Higgs sector all masses are obtained from the effective potential where the one-loop corrections to the quartic couplings are included. The mass of the SM-like Higgs is then computed with radiative corrections to the quartic couplings considered at the one-loop level. The SM-like Higgs mass easily satisfies the experimental bound $m_{h^0} > 114.5$ GeV for $M_c > 6.5$ TeV. The LSP is the Higgsino-like with mass $\sim \mu$. Electroweak precision observables also put lower bounds on M_c (see e.g. Ref. 5). For the particularly chosen model the $\chi^2(M_c)$ distribution has a minimum around $M_c \simeq 10.5$ TeV and one deduces $M_c > 4.9$ TeV at 95% c.l.

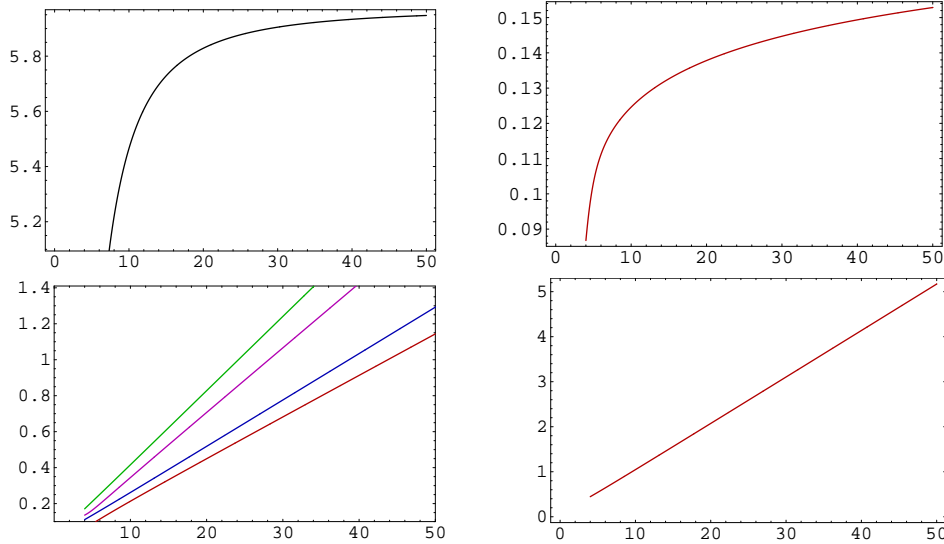


Fig. 1. Predictions for the case $\omega = 0.45$, $\tilde{\omega} = 0.35$, $M = 1.65M_c$ as a function of the compactification scale. Upper left panel: $\tan\beta$. Upper right panel: the SM-like Higgs mass m_h . Lower left panel, from top to bottom the lines correspond to the masses of: left-handed sleptons $m_{\tilde{e}_L}$, heavy neutral Higgs (with a mass approximately equal to the pseudoscalar mass) $m_H \simeq m_A$, right-handed sleptons $m_{\tilde{e}_R}$ and neutralinos $m_{\chi^0} \simeq \mu$. Lower right panel: the squark masses $m_{\tilde{q}}$. All masses are in TeV.

The most salient prediction of this model is the ratio of the squark and slepton masses. As the Higgs sector is effectively localized, to leading order in perturbation theory these masses are generated entirely by gauge/gaugino loops. They are finite and calculable due to the nonlocal nature of SS breaking. The leading contribution

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to the stop and sbottom masses were already given in Eq. (23), adding all gauge contributions one finds

$$(m_{\tilde{q}_L}, m_{\tilde{u}_R}, m_{\tilde{d}_R}, m_{\tilde{\ell}_L}, m_{\tilde{e}_R}) = (0.110, 0.103, 0.102, 0.042, 0.025)\sqrt{f(\omega)}M_c. \quad (31)$$

The ratios of the masses are independent on M_c and ω , and are simply calculated from the couplings and group theoretic invariants. Recall that similar relations are known from gauge mediation models (see Ref. 30 for a review). There however scalar masses are generated at the two loop level and hence different ratios apply. Depending on the size of the messenger scale and other details of the model, these relations can receive important corrections from renormalization group (RG) running. In our case we expect RG effects to be small, as the high scale (M_c) is at most two orders of magnitude above the low scale (the actual masses). Other small corrections are expected from higher loops as well as EWSB.^g

Finally in the considered class of models where the neutralino is the LSP and R -parity is conserved the lightest neutralino is the candidate to Cold Dark Matter. The prediction of $\Omega_{\tilde{\chi}^0}h^2$ can be obtained using the DarkSUSY package³¹ and can also be approximated by the expression³²

$$\Omega_{\tilde{\chi}^0}h^2 \simeq 0.09 (\mu/\text{TeV})^2 \quad (32)$$

In the particular model of Fig. 1 the prediction of $\Omega_{\tilde{\chi}^0}h^2$ is given in Fig. 2

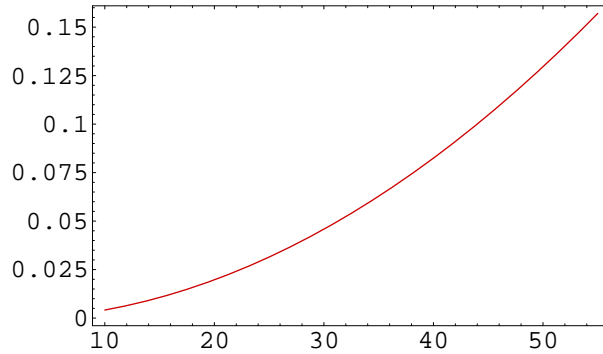


Fig. 2. $\Omega_{\tilde{\chi}^0}h^2$ as a function of M_c (in TeV) for the model presented in Fig. 1.

Recent WMAP results²⁹ imply that $0.114 < \Omega_{\tilde{\chi}^0}h^2 < 0.134$. As one can see from Fig. 2 this range in $\Omega_{\tilde{\chi}^0}h^2$ points towards the range $49 \text{ TeV} < M_c < 53 \text{ TeV}$.^h Then for a value of $M_c \sim 50 \text{ TeV}$ the density of Dark Matter agrees with the

^gThere are also one-loop generated A terms.^{3,4} The effect on the stop mass is approximately $(\Delta^A m_{\tilde{t}}/m_{\tilde{t}})^2 \sim v/M_c$.

^hSuch large values for M_c require an increased fine-tuning.

recent results obtained from WMAP. Notice that for such large values of M_c the neutralinos are almost Dirac particles. However the non-Dirac character is spoiled by $\mathcal{O}(m_W/M_{1/2})m_W \sim 300$ MeV which is enough to avoid the strong limits on Dirac fermions that put a lower bound on the non-Diracity around 100 KeV.^{33,34} On the other hand the WMAP range for M_c implies, in the gravitational sector, gravitino masses $m_{3/2} \gtrsim 10$ TeV (depending on the value of the SS parameter ω) are such that gravitinos decay early enough to avoid cosmological troubles and thus solving the longstanding cosmological gravitino problem.³⁵

5. Conclusions

The interplay of Extra Dimensions and supersymmetry can help to construct realistic models of electroweak symmetry breaking. Supersymmetry breaking by the SS-mechanism leads to finite and calculable soft mass terms, controlled by the compactification scale. In order to generate strong enough supersymmetry breaking in the Higgs sector, either the top/stop sector or the Higgs sector must propagate in the bulk of the Extra Dimension, although they may be quasi-localized to a high degree due to the presence of bulk mass terms. We have presented a model that uses a quasilocalized Higgs to successfully trigger EWSB. Both μ -term and tachyonic soft mass terms are present at tree level and are seen to have a geometric origin though the boundary conditions. Moreover, the leading tree-level, one- and two-loop contributions to the soft squared masses can naturally be of the same order, leading to a rather modest fine-tuning in this model. The characteristics of the model are a compactification scale in the multi TeV range ($M_c \gtrsim 5$ TeV), heavy (universal) gauginos $M_{1/2} = \omega M_c$ (with $\omega = .25 - .5$) and a characteristic and calculable ratio of squark/slepton masses, Eq. (31). The LSP is a neutralino (which is almost pure Higgsino). It is a good DM candidate for higher values of $M_c \sim 50$ TeV.

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